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Effect of external parameters on interface-LO-phonon amplification in quantum wire

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Abstract. Under a laser field, the interface-LO-phonon amplification effect in a quantum wire is presented in this paper. The rate of change of the phonon population is calculated, and the amplification condition is given. It is shown that in the low temperature region, there are lower and upper thresholds of the laser field strength for achieving phonon amplification in a quantum wire.

The process of cooling of the hot electrons in a quantum wire, excited by a laser field, has been the subject of intense experimental and theoretical investigation in recent years [1–5]. It has been shown that electron–phonon scattering in quantum wires not only causes instability of the electron system, but also causes instability of the phonon system. For a particular case of this relaxation process, the electron–phonon scattering can cause the excitation of higher harmonics and the phonon amplification under an intense laser field [6, 7]. For the unstable phonon system would severely affect the device performance [8, 9]; it is necessary for us to investigate the process of phonon amplification. In a quantum wire, the electrons are localized in the wire and form a quasi-one-dimensional electron gas while the interface phonons are localized in the interface, so that the electron–interface-phonon scattering is responsible for the instability of the interface. Since the electron–optical-phonon scattering rate is larger than the electron–acoustic-phonon rate [2], only the process of interface-optical-phonon amplification is considered in this paper. In the wire, the electrons transfer energy obtained from the laser field to the LO phonons by the electron–phonon interaction. The phonon amplification resulting from the electron–LO phonon interaction under a laser field leads to a phonon flux along the wire. The focus of this work is to make it clear that how the external parameters, such as the laser field strength and the temperature, affect the phonon amplification in a quantum wire.

In the following section, we first calculate the matrix element of the electron–interface-LO-phonon scattering, then give the rate of change of the phonon population and finally discuss the condition of phonon amplification.

We consider a simple model for a quantum wire [1], in which a quasi-two-dimensional electron gas formed in a heterostructure is confined by narrow gates or split gates, and the electrons are free along only one direction. We assume that the heterointerface is normal to the z axis and the confinement in the y axis direction is characterized by a parabolic potential of frequency Ω_y. For the confinement potentials along the z axis, we take a triangular potential well. The dimensions of the hetero-interface are assumed to be $S = L_x L_y$. A polarized laser irradiates the sample normal to the interface, with its polarization along the x axis and its strength expressed as a vector potential

$$\vec{A}(t) = A_0 \vec{i} \cos(\omega t) \tag{1}$$

neglecting its space distribution.

To calculate the matrix elements for the electron–interface-LO-phonon scattering, we solve the Schrodinger equation for the electron wave function $\Psi(\vec{r}, t)$

$$\mathrm{i}\hbar \frac{\partial}{\partial t}\Psi(\vec{r}, t) = \frac{1}{2m}\left[\vec{P} - \frac{e}{c}\vec{A}(t)\right]^2 \Psi(\vec{r}, t) + \frac{1}{2}\Omega_y^2 y^2 \Psi(\vec{r}, t) - eFz\Psi(\vec{r}, t) \tag{2}$$

where F is a constant related to the concentration of the modulation doping [10]. The solution of the Schrodinger equation for an electron in the fields is

$$|k, l, n\rangle = \Psi_{k,l,n}(\vec{r}, t) = \left(\frac{1}{2^l l!}\sqrt{\frac{m\Omega_y}{\hbar\pi}}\right)^{1/2} \frac{1}{\sqrt{L_x}} \mathrm{e}^{\mathrm{i}kx} H_l\left(\sqrt{\frac{m\Omega_y}{\hbar}}y\right) \tilde{A}_n(z)$$
$$\times \exp\left(-\frac{m\Omega_y}{2\hbar} y^2 - \frac{\mathrm{i}}{\hbar}\int_0^t \left\{ E_{l,n} + \frac{1}{2m}\left(\hbar k - \frac{e}{c}A(t)\right)^2 \right\} \mathrm{d}t \right) \tag{3}$$

where $H_l(\sqrt{m\Omega_y/\hbar}\,y)$ are Hermite polynomials, and $\tilde{A}_n(z)$ is the normalized Airy function. The energy spectrum is

$$E_{k,l,n} = \frac{\hbar^2 k^2}{2m} + (l + \tfrac{1}{2})\hbar\Omega_y + eFdX_n \tag{4}$$

where X_n is the sth zero point of the Airy function, and $d = (\hbar^2/2meF)^{1/3}$.

We assume that the interface-LO phonons in the quantum wire still have the characteristics of the interfacial phonon, with field operator [10]

$$\phi(q_\parallel, \omega_I; \vec{r}_\parallel, t) = C\mathrm{e}^{q_\parallel z}\mathrm{e}^{\mathrm{i}(\vec{q}_\parallel \vec{r}_\parallel - \omega_I t)} \tag{5}$$

where $\vec{q}_\parallel$ is the phonon wave vector paralleled to the interface, i.e.,

$$\vec{q}_\parallel = q_x \vec{i} + q_y \vec{j} \tag{6}$$

ω_I are the frequencies of interface phonons neglecting phonon dispersion, and

$$C = -\left(\frac{e^2\hbar}{2L_x L_y \varepsilon_0 q_\parallel \omega_I}\right)^{1/2} \frac{(\omega_{T_1}^2 - w_I^2)(\omega_{T_2}^2 - \omega_I^2)}{[\varepsilon_{\infty_1}(\omega_{L_1}^2 - \omega_{T_1}^2)(\omega_{T_2}^2 - \omega_I^2) + \varepsilon_{\infty_2}(\omega_{L_2}^2 - \omega_{T_2}^2)(\omega_{T_1}^2 - \omega_I^2)]^{1/2}}. \tag{7}$$

Here ε_0 is the low frequency dielectric constant, $\varepsilon_{\infty_\mu}(\mu = 1, 2)$ are the optical dielectric constants on both sides of the interface and ω_{L_μ} and $\omega_{T_\mu}(\mu = 1, 2)$ the LO and TO bulk phonon frequencies, respectively.

To obtain analytical results, a single-band approximation for the electrons and a systematic Airy function are adopted [12]. Considering the electron–phonon interaction as a perturbation, one can calculate the transition matrix elements from initial state $|k, 0, 0\rangle$ to final state $|k', 0, 0\rangle$ and then calculate the corresponding transition probability per unit time $T(k, 0, 0)$ for electron transitions from $|k, 0, 0\rangle$ to all the $|k', 0, 0\rangle$, that is

$$T(k, 0, 0) = \frac{128\pi b^6 |C|^2}{\hbar(2b + q_\parallel)^6} \mathrm{e}^{-\frac{2Q_y^2}{\alpha^2}} \sum_{\nu=-\infty}^{+\infty} \left| J_\nu\left(\frac{\Lambda}{\hbar\omega}\right)\right|^2 \delta(E_{k+q_x,0,0} - E_{k,0,0} - \hbar\nu\omega - \hbar\omega_I) \tag{8}$$

where

$$\alpha = \left(\frac{m\Omega_y}{2\hbar}\right)^{1/2} \tag{9}$$

$$b = \left(\frac{2meF}{\hbar^2}\right)^{1/2} \frac{1}{X_0 - 1} \tag{10}$$

and

$$\Lambda = \frac{\hbar e q_x A_0}{mc} = \frac{\hbar e q_x E_0}{m\omega} \tag{11}$$

where E_0 is the field strength of the laser field. Λ is assumed to be large enough that the following approximation can be employed [13]:

$$\sum_{\nu} \left| J_\nu \left(\frac{\Lambda}{\hbar\omega} \right) \right|^2 \delta(E - \hbar\nu\omega) = \frac{1}{2}[\delta(E - \Lambda) + \delta(E + \Lambda)]. \tag{12}$$

If $\Lambda > \hbar\omega_I$ and the laser–photon absorption process dominates, we can then neglect the contribution of the first δ function in equation (12). The condition $\Lambda > \hbar\omega_I$ is also expressed by saying that the electron drift velocity along the wire is larger than the phase velocity of the interface phonons. The laser field gives the electron the drift velocity, and then the electron–phonon interaction results in phonon excitation.

Considering the process of phonon absorption and emission simultaneously in a similar way as in our previous calculations [6, 7], the kinetic equation for the phonon population is given by

$$\frac{\mathrm{d}N_{q_x,q_y}}{\mathrm{d}t} = \gamma_{q_x,q_y} N_{q_x,q_y} \tag{13}$$

where γ_{q_x,q_y} is the rate of change of the phonon population. This can be expressed as

$$\gamma_{q_x,q_y} = \sum_k T(k,0,0)[f(k+q_x,0,0) - f(k,0,0)] \tag{14}$$

where $f(k,0,0)$ is the Fermi distribution function, namely

$$f(k,0,0) = \frac{1}{\exp\left\{ \frac{\hbar^2 k^2}{2mkT} + \frac{\hbar\Omega_y}{2kT} + \frac{eFdX_0}{kT} - \frac{\mu}{kT} \right\} + 1} \tag{15}$$

where μ is the chemical potential. Under a laser field and when the electron drift velocity exceeds the phase velocity of the phonons, a detailed calculation yields

$$\begin{aligned}
\gamma_{q_x,q_y} = {} & \frac{64mb^6|C|^2L_x}{\hbar^3 q_x(2b+q_\parallel)^6} \exp\left\{-\frac{2q_y^2}{\alpha^2}\right\} \int \mathrm{d}k\delta\left[k + \frac{m}{\hbar^2 q_x}\left(\frac{\hbar^2 q_x^2}{2m} + \Lambda - \hbar\omega_I\right)\right] \\
& \times[f(k+q_x,0,0) - f(k,0,0)] = \frac{64mb^6|C|^2L_x}{\hbar^3 q_x(2b+q_\parallel)^6} \exp\left\{-\frac{2q_y^2}{\alpha^2}\right\} \\
& \times\left[\frac{1}{\exp\left\{\frac{m}{2\hbar^2 q_x^2 kT}\left(\frac{\hbar^2 q_x^2}{2m} + \Lambda - \hbar\omega_I\right)^2 - \frac{\Lambda - \hbar\omega_I}{kT} + \frac{\frac{1}{2}\hbar\Omega_y + eFdX_0 - \mu}{kt}\right\} + 1}\right. \\
& \left. - \frac{1}{\exp\left\{\frac{m}{2\hbar^2 q_x^2 kT}\left(\frac{\hbar^2 q_x^2}{2m} + \Lambda - \hbar\omega_I\right)^2 + \frac{\frac{1}{2}\hbar\Omega_y + eFdX_0 - \mu}{kT}\right\} + 1}\right].
\end{aligned} \tag{16}$$

This shows that only when $\Lambda > \hbar\omega_I$ does the rate γ_{q_x,q_y} become positive and optical-phonon amplification is achieved. Equation (16) also shows that the rate of change of the phonon population is sensitive to the phonon wave vector $\vec{q}_\parallel$, and it has a maximum value if $q_x = q_x^l$ (the lower limit of q_x), and $q_y = 0$. At a fixed value q_y, the thinner the quantum wire (Ω_y taken as larger value), the larger the rate γ_{q_x,q_y} will be. This demonstrates that the electron–phonon interaction in the quantum wire is stronger than that in the quantum well.

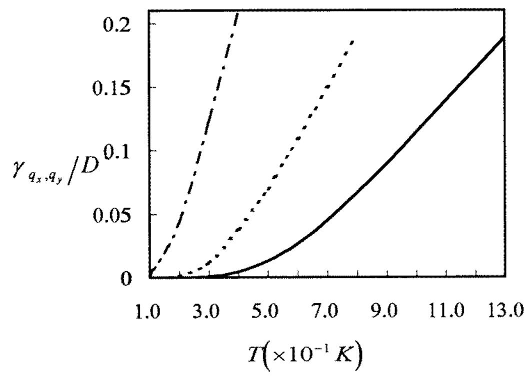

Figure 1. The rate γ_{q_x,q_y} as a function of the temperature with different field strengths. Dashed, dot–dashed, and solid lines for laser field strength $E_0 = 4.90$ V cm^{-1}, $E_0 = 4.95$ V cm^{-1} and $E_0 = 5.00$ V cm^{-1}, respectively.

When the chemical potential is close to the energy of the electron ground state, the following condition is satisfied

$$\frac{m}{2\hbar^2 q_x^2}\left(\frac{\hbar^2 q_x^2}{2m} + \Lambda - \hbar\omega_I\right)^2 + \frac{1}{2}\hbar\Omega_y + eFdX_0 > \mu + \Lambda - \hbar\omega_I. \tag{17}$$

In this condition and at low temperature, equation (16) can be approximated as

$$\begin{aligned}\gamma_{q_x,q_y} = \frac{64mb^6|C|^2 L_x}{\hbar^3 q_x(2b+q_\parallel)^6}\exp\left\{-\frac{2q_y^2}{\alpha^2}\right\}\Bigg[\exp\Bigg\{-\frac{m}{2\hbar^2 q_x^2 kT}\left(\frac{\hbar^2 q_x^2}{2m} + \Lambda - \hbar\omega_I\right)^2 \\ + \frac{\Lambda - \hbar\omega_I}{kT} - \frac{\frac{1}{2}\hbar\Omega_y + eFdX_0 - \mu}{kT}\Bigg\} \\ - \exp\left\{-\frac{m}{2\hbar^2 q_x^2 kT}\left(\frac{\hbar^2 q_x^2}{2m} + \Lambda - \hbar\omega_I\right)^2 - \frac{\frac{1}{2}\hbar\Omega_y + eFdX_0 - \mu}{kT}\right\}\Bigg].\end{aligned} \tag{18}$$

Since the electron number in the quantum wire is

$$N = \frac{L_x}{\pi}\int_{-\infty}^{+\infty} \mathrm{d}k \exp\left\{-\frac{\hbar^2 k^2}{2mkT} - \frac{\frac{1}{2}\hbar\Omega_y + eFdX_0 - \mu}{kT}\right\} \tag{19}$$

we obtain

$$\exp\left\{-\frac{\frac{1}{2}\hbar\Omega_y + eFdX_0 - \mu}{kT}\right\} = \frac{N}{L_x}\left(\frac{\pi\hbar^2}{2m}\right)^{1/2}. \tag{20}$$

Equation (20) is substituted into equation (18); we obtain

$$\begin{aligned}\gamma_{q_x,q_y} = D\Bigg[\exp\left\{-\frac{m}{2\hbar^2 q_x^2 kT}\left(\frac{\hbar^2 q_x^2}{2m} + \Lambda - \hbar\omega_I\right)^2 + \frac{\Lambda - \hbar\omega_I}{kT}\right\} \\ - \exp\left\{-\frac{m}{2\hbar^2 q_x^2 kT}\left(\frac{\hbar^2 q_x^2}{2m} + \Lambda - \hbar\omega_I\right)^2\right\}\Bigg]\end{aligned} \tag{21}$$

where

$$D = \frac{64Nb^6|C|^2}{\hbar^3 q_x(2b+q_\parallel)^6}\left(\frac{\pi m\hbar^2}{2}\right)^{1/2}\exp\left\{-\frac{2q_y^2}{\alpha^2}\right\}. \tag{22}$$

The rate γ_{q_x,q_y} at low temperature is given as a function of temperature. To analyse an external field how to affect phonon amplification, we take $Al_xGa_{1-x}As$–GaAs wire for an example to calculate the rate γ_{q_x,q_y} for different field strengths. The parameters are chosen as $m = 0.068m_0$ (m_0 for the electron static mass), $\omega_I = 6.871 \times 10^{13}$ s^{-1}, $\omega = 4.82 \times 10^{13}$ s^{-1} and $q_x = 1.0 \times 10^6$ cm^{-1}. The calculated results are plotted in figure 1. It shows that at low temperature, only a laser field with adequate strength can induce the amplification of the phonons with certain one wavevector q_x. It is because only when the electron-drift energy Λ is comparable with the thermal-motion energy of the phonons do they couple each other efficiently. As the electron-drift energy is proportional to the field strength E_0, and the thermal-motion energy of the phonons is proportional to $\hbar\omega_I e^{-\frac{\hbar\omega_I}{kT}}$ at low temperature [14], the phonon amplification induced by the weaker laser field will be achieved at low temperature.

In conclusion, we have calculated the rate of change of the interface-phonon population of a quantum wire under laser excitation. The results show that an external field with adequate strength can induce phonon amplification in a quantum wire. When the phonon wave vector is small, the rate of change of the phonon population is large. Only when q_x is positive, $\gamma_{q_x,q_y} > 0$, will the phonon population increase with time. Therefore, it is possible for us to obtain a quasi-travelling wave of amplified phonons. The propagating direction is the same as the direction of the vector potential. Our results also show that the electron–phonon interaction in a quantum wire is stronger than that in the quantum well.

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